

Sherlock Holmes and the Raven-Paradox

A contemporary of Sherlock Holmes, Carl Gustav Hempel (1902-1997) of the Vienna Circle, posed a famous problem in logics. It was often classified as a paradox,¹ though not by Hempel himself:² Suppose the colour of raven is to be investigated and the first 100 raven encountered were black. Then we may be inclined to hypothesize that the relation **R a B** is true, "All raven are black". By logical contra-position we obtain the relation: **nB a nR** "All not-black things are not raven". This should be exactly equivalent to **R a B**. Then, however, the observation of a yellow submarine, a **nB** which is also a **nR**, tends to support our working hypothesis, even though a yellow submarine seems irrelevant for the colour of raven. What is wrong?

Well, suppose there is a piano keyboard with 13 keys (one octave). Some keys are long, others short, some are black, others not, and detectives of the Conan-Doyle-Agency are to investigate their colour-length relation. Sherlock Holmes, shrewdly concentrating on the short keys, uses first a ruler, then his pocket spectrometer. Halfway through the job he pronounces his working hypothesis: **S a B** "All short keys are black". Now Holmes must record the colour of all 5 short keys before he can definitely conclude, based on "total evidence",³ that **S a B** is true. A single case of **S i nB** "A short key is not black" would falsify his hypothesis. Dr. Watson, sceptically choosing the other approach, deals with the not-black keys, using first a spectrometer, then a ruler. After finding 8 not-black keys and checking their length, he first reasons that **nB a nS**, as he did not find a case of **nB i S**. In a well-worn copy of the "Handbook of Logics" he looks up the contra-position and concludes that Holmes is right: **S a B** is true. At this point Holmes is already playing (Romance in F-major) on his beloved Amati.

Information⁴ counts the yes-or-no-possibilities (bits) to be decided upon until certainty is reached.⁵ As Holmes used to say: "Eliminate the impossible and the remainder, however unlikely, must be the truth."⁶ It took Holmes 5 bits to arrive at **S a B**. Watson needed 8 bits to arrive at **nB a nS**. Given only 13 keys in total, it is easy to see that the observation of short or of not-black keys must lead to the same conclusion. Watson's way is OK too, but lacks efficiency.

What about the raven? Holmes reasons as follows: Say #R is the total number of raven and #T the total number of not-black things. While we do not know these numbers exactly, it is safe to say that #T >> #R. Thus the number of decisions, required to reach "total evidence", is much smaller when we concentrate on raven rather than on not-black things. This quantitative consideration explains the contra-intuitive flavour of the raven paradox. However, it is not a paradox, my dear Watson. It is a case of differing efficiency.

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- 1 e.g. Taleb, Nassim Nicholas. 2007. *The Black Swan. The impact of the highly improbable*. New York: Random House: page 84
 - 2 Hempel, Carl Gustav. 1945. Studies in the Logic of Confirmation I. *Mind* 54 (213):1-26. Hempel, Carl Gustav. 1945. Studies in the Logic of Confirmation II. *Mind* 54 (214):97-121. A quantitative solution was proposed already 1940 by Mrs Hosiasson-Lindenbaum in her "Theory of Degrees of Confirmation", see Hosiasson-Lindenbaum, Janina. 1940. On Confirmation. *The Journal of Symbolic Logic* 5 (4):133-148. Nevertheless, many of the more recent authors classify the problem as an unsolved paradox. As Sherlock Holmes shows below in simple words, this is not justified.
 - 3 Hempel, Carl Gustav. 1960. Inductive inconsistencies. *Synthese* 12:439-469: 450.
 - 4 the amount of information measured in $\log_2(\text{bit})$
 - 5 Adams, Frederick. 1999. Information theory. In *The Cambridge Dictionary of Philosophy*, ed. Robert Audi, 435-437. Cambridge, UK: Cambridge University Press.
 - 6 Conan Doyle, Sir Arthur. 1890. The sign of the four. London: Lippincott's Monthly Magazine.