Sherlock Holmes and the Raven-Paradox

A contemporary of Sherlock Holmes, Carl Gustav Hempel (1902-1997) of the Vienna Circle, posed a famous problem in logics. It was often classified as a paradox,[1] though not by Hempel himself:[2] Suppose the colour of raven is to be investigated and the first 100 raven encountered turn out to be black. Then we may be inclined to hypothesize that the relation:

R a B "All raven are black".

is true. By logical contra-position we obtain the relation:

nB a nR "All not-black things are not raven".

This should be exactly equivalent to **R a B**. Then, however, the observation of a yellow submarine, which is a **nB** and, therefore, also a **nR**, tends to support our working hypothesis, even though a yellow submarine seems irrelevant for the colour of raven. What is wrong?

Sherlock Holmes was called. I am to count decisions until total evidence is reached, says he. Suppose #R is the number of raven and #nB the number of not-black things. While we do not know these numbers exactly, it is safe to say that

#nB >> #R.

Thus the number of decisions required to reach "total evidence" is much smaller when we concentrate on raven rather than on not-black things. To concentrate on the smaller number is more efficient. This explains the paradox flavour of the raven-and-submarine problem. However, it is not a paradox, my dear Watson. It is a case of differing efficiency. But when, Holmes continued, have we reached total evidence?

For fun, lets try another setting. Suppose there is a piano keyboard with 13 keys (one octave). Some keys are long, others short, some are black, others not, and detectives of the Conan-Doyle-Agency once morei are to investigate their colour-length relation. Sherlock Holmes, shrewdly concentrating on the short keys, uses first a ruler, then his pocket spectrometer. Halfway through the job he pronounces his working hypothesis:

S a B "All short keys are black".

Now Holmes must record the colour of all 5 short keys before he can definitely conclude, based on "total evidence",[3] that **S a B** is true. A single case of **S i nB** "A short key is not black" would falsify his hypothesis.

1 e.g. Taleb, Nassim Nicholas. 2007. *The Black Swan. The impact of the highly improbable*. New York: Random House: page 84

2 Hempel, Carl Gustav. 1945. Studies in the Logic of Confirmation I. Mind 54 (213):1-26. Hempel, Carl Gustav. 1945. Studies in the Logic of Confirmation II. *Mind* 54 (214):97-121. A quantitative solution was proposed already 1940 by Mrs Hosiasson-Lindenbaum in her "Theory of Degrees of Confirmation", see Hosiasson-Lindenbaum, Janina. 1940. On Confirmation. *The Journal of Symbolic Logic* 5 (4):133-148. Nevertheless, many of the more recent authors classify the problem as an unsolved paradox. As Sherlock Holmes shows below in simple words, this is not justified.

3 Hempel, Carl Gustav. 1960. Inductive inconsistencies. *Synthese* 12:439-469: 450.

Dr. Watson, sceptically choosing another approach, deals with the not-black keys, using first a spectrometer, then a ruler. After finding 8 not-black keys and checking their length, he reasons that

nB a nS “Not-black keys are not short”

as he did not find a case of **nB i S**. In a well-worn copy of the "Handbook of Logics" he looks up the contra-position and concludes that Holmes is right: **S a B** is true. At this point Holmes is already playing the Romance in F-major on his beloved Amati.

Information counts the decisions needed to reach certainty.[4] As Holmes used to say: "Eliminate the impossible and the remainder, however unlikely, must be the truth."[5] It took Holmes 5 decisions to arrive at **S a B**. Watson needed 8 to arrive at **nB a nS.** Given only 13 keys in total, it is easy to see that the count of short keys or of not-black keys must lead to the same conclusion, though with different efficiency. Watson's way is OK too, but less efficient.

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4 Adams, Frederick. 1999. Information theory. In *The Cambridge Dictionary of Philosophy*, ed. Robert Audi, 435-437. Cambridge, UK: Cambridge University Press.
5 Conan Doyle, Sir Arthur. 1890. The sign of the four. London: Lippincott's Monthly Magazine.